Tutorial 3

Microeconomics 3, Part 2

Question 1

Consider an economy with $I = 2$ consumers and $L = 2$ goods. There is a single firm whose only technology is free disposal. The consumers' utility functions over both goods are given by:

$$u_1(x_{11}, x_{21}) = 2\sqrt{x_{11}} + x_{21}$$
$$u_2(x_{12}, x_{22}) = x_{12} + bx_{22}$$

where $b \geq 0$ is a parameter. The consumption set for both consumers is $X_i = \mathbb{R}^2_+$. The aggregate endowment is given by $\bar{\boldsymbol{\omega}} = (1, 1)$.

(i) Consider the allocation $\boldsymbol{x}_1 = (0, 1)$ and $\boldsymbol{x}_2 = (1, 0)$. Is this allocation Pareto optimal? If the answer depends on the value of b, give different cases. If in a particular range of values of b it is not Pareto optimal, give an example allocation (that may depend on the value of b) that is Pareto improving.

(ii) Can the allocation in (i) arise in a competitive equilibrium for some initial endowments $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$? Again, consider different cases for the value of b.

(iii) Explain the relevance (or lack thereof) of the second welfare theorem in your answer to (ii).

Question 2

Consider an exchange economy with two commodities and two households. Household 1 owns six units of commodity 1 and two units of commodity 2, whereas household 2 owns ten units of commodity 1 and two units of commodity 2. Both households have the same preferences, represented by the utility function $u(x, y) = \sqrt{x} + 2\sqrt{y}$, where x is the consumption of commodity 1 and y is the consumption of commodity 2. Let $z(p)$ be the aggregate excess demand at price vector p.

(i) Derive $z(p)$ for any price vector p.

(ii) Show that z is bounded from below.

(iii) Show that z satisfies Walras' law.

(iv) Show that z satisfies homogeneity of degree zero.

(v) Show that z satisfies desirability, i.e., if the price of a commodity converges to zero, its excess demand goes to infinity.

(vi) Show that z satisfies continuity if all prices are positive.

(vii) Conclude that there exists a price vector p^* such that $z_1(p^*) = 0$ and $z_2(p^*) = 0$. Calculate this price vector (with sum of the prices equal to 1) and check that it is indeed a Walrasian equilibrium price vector.

(viii) Is z gross substitutable?

(ix) Conclude that this economy has only one Walrasian equilibrium (up to price normalization).

Question 3

Consider a pure exchange economy with L goods and I consumers with preferences $\succeq_i$ over $X_i = \mathbb{R}_+^L$ that admit an aggregate demand function $\boldsymbol{z}(\boldsymbol{p})$ that is continuous over all $\boldsymbol{p} \in \mathbb{R}_+^L$ and satisfies:

- $\boldsymbol{z}(\boldsymbol{p}) = \boldsymbol{z}(\alpha \boldsymbol{p})$ for all $\alpha > 0$ (homogeneity of degree zero).
- $\boldsymbol{p} \cdot \boldsymbol{z}(\boldsymbol{p}) \leq 0$ for all $\boldsymbol{p} \in \mathbb{R}_+^L$.

The distribution of the endowment vector satisfies $\boldsymbol{\omega}_i \gg \mathbf{0}$ for all i. There is a single firm whose production set is

$$Y_1 = \left\{ \boldsymbol{y}_1 \in \mathbb{R}^L : \boldsymbol{y_1} \leq \mathbf{0} \right\}$$

Each consumer i is entitled to a share $\theta_{i1} = \frac{1}{I}$ of the firm's profits.

Define the following function $f : \Delta \to \Delta$:

$$\{f_\ell(\boldsymbol{p})\}_{\ell=1}^{L} = \left\{ \frac{p_\ell + \max\{0, z_\ell(\boldsymbol{p})\}}{1 + \sum_{k=1}^{K} \max\{0, z_k(\boldsymbol{p})\}} \right\}_{\ell=1}^{L}$$

where:

$$\Delta = \left\{ \boldsymbol{p} \in \mathbb{R}_+^L : \sum_{\ell=1}^{L} p_\ell = 1 \right\}$$

(i) [**2 points**] Show that Δ is convex.

(ii) [**2 points**] Show that Δ is compact.

(iii) [**2 points**] Show that if $\boldsymbol{p} \in \Delta$, then $f(\boldsymbol{p}) \in \Delta$.

(iv) [**4 points**] Prove that there exists a price vector $\boldsymbol{p}^\star \geq \mathbf{0}$, $\boldsymbol{p}^\star \neq \mathbf{0}$ that satisfies $\boldsymbol{z}(\boldsymbol{p}^\star) \leq \mathbf{0}$.

(v) [**4 points**] If there is a price vector $\boldsymbol{p}^\star \geq \mathbf{0}$, $\boldsymbol{p}^\star \neq \mathbf{0}$ that satisfies $\boldsymbol{z}(\boldsymbol{p}^\star) \leq \mathbf{0}$, is it a Walrasian equilibrium price vector?